B.Sc - Part I
PAPER - I

Page No.:
Date: / /

Integration of some special trigonometric functions.

I. If n is an odd positive integer, the integrals $\int \sin^n x\, dx$ and $\int \cos^n x\, dx$ can be performed by substituting $\cos x = t$ and $\sin x = t$ respectively.

Example

Integrate $\int \sin^7 x\, dx$

Solⁿ Here $I = \int \sin^6 x \sin x\, dx$

$$= \int (1-\cos^2 x)^3 \sin x\, dx$$

Put $\cos x = t$ $\therefore -\sin x\, dx = dt$

$$\therefore I = -\int (1-t^2)^3 dt$$

$$= -\int (1 - 3t^2 + 3t^4 - t^6)\, dt$$

Page No.:
Date: / /

$= -\int dt + 3\int t^2 dt - 3\int t^4 dt + \int t^6 dt$

$= -t + t^2 - \frac{3}{5} t^5 + \frac{t^7}{7} + C$

$= -\cos x + \cos^2 x - \frac{3}{5}\cos^5 x + \frac{\cos^7 x}{7}$

$+ C.$

II If n is an even positive integer and small, the integrals $\int \sin^n x\, dx$ and $\int \cos^n x\, dx$ can be worked out by expressing the integrand in term of multiple angles.

Example.

1 Integrate

$\int \sin^4 x\, dx.$

Sol^n we have $\cos 2x = 1 - 2\sin^2 x$

$\therefore \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\therefore \sin^4 x = \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$

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B.Sc Part I 02-01-20

$$\therefore I = \frac{1}{4}\int\left[\frac{3}{2} - 2\cos 2x + \frac{\cos 4x}{2}\right]dx$$

$$= \frac{1}{4}\cdot\frac{3}{2}\int dx - \frac{1}{2}\int \cos 2x$$

$$+ \frac{1}{8}\int \cos 4x\, dx$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x$$

$$+ C$$

where C is the constant of integ-ration.